

How to Read a Math Textbook Correctly

One of the things many students have a difficult time learning in college is how to read a math textbook. In fact, the better-prepared you are in math, the harder it is to learn!

If you're anything like I was, you kind of resent the implication that you don't know how to read. Of course you know how to read! I wouldn't be writing this document for you if I believed otherwise.

So then what do I mean? I mean that proof-based math courses are among the most intellectually rigorous, serious courses available in college. Counterintuitively, the books for these courses get thinner and thinner the higher-level the course! This is because the giant tomes in earlier courses provide you with every last detail of every last argument and example, as they try to train you in that particular topic.

Books for later courses are trying to train you in their topic, too. But they're going to start leaving out details. Lots of details. Details of calculations, steps in logical arguments, or even entire proofs will be omitted! This doesn't mean those details are unimportant: to the contrary, it is *crucial* that they be diligently supplied – **by the reader**.

This isn't done out of cruelty, laziness or frugality. It's because the act of reading a math textbook line by line, parsing every symbol, every mathematical phrase, every logical maneuver with a fine-toothed-comb, pen in hand, writing out all the details? It's a great way to learn. In fact, most mathematicians would tell you it's the only way.

Why not just go to lecture and passively be told true things? It's worked so far, right?

Because each mathematical topic is like every other human skill: it's acquired by modeling, practice and feedback. The textbook, when read correctly, provides all of these. So here's how to read it right.

Specifically for Math 310, you'll need to spend a lot of time focused on the examples. Why are the proofs constructed the way they are? Would another structure work just as well? How does this example relate to the ones done in class/homework/group work? How does each step follow from the previous step? Each proof is like a puzzle: it fits together just exactly right. Your job is to understand that structure in every detail and at every scale. Step 7 on the next page is especially important for this.

My recommended process is on the next page.

1. Schedule the time. To start, arrange thirty minutes interruption-free. No phone, no laptop, no conversations. Just you and the book, maybe some non-distracting music. Length of time will increase as your stamina for this exercise increases.
2. Identify what material you intend to read. Be aware that you can easily spend your half-hour on two pages or less!
3. Get out pen and paper. You aren't reading if you aren't writing.
4. Skim the section you're reading.
 - In this first skim, you aren't trying to follow every detail of every argument. Think big picture.
 - You should have the following question in mind: "What is the point of this section? What is our main goal here?" Write down what you believe is the main idea.
 - Basically everything in the section will be somehow in service of the main goal. From your skim, can you identify the logical structure? How does Lemma 2 help establish Theorem 4? Draw a picture of the dependencies.
5. Now that you have an overall idea of the structure of the section, it's time to get to business. Go to the first definition. Write it down. Parse every symbol, every word, every line of it.
6. Say we're defining, for example, a "*ring*." Your focus should not be on why anybody might care about rings. It should be on understanding every part of the definition: what makes something a ring? What fundamental property or structure are we defining here? Think first about *what*, not *why*. The why and how come soon enough.
7. Usually, next comes some example involving this new definition.
 - Read the statement of the example several times, one symbol at a time, one clause at a time. Rephrase it in your own words. What's the gist of it? What does it *say*?
 - Now skim the proof, checking for structure: are they using a proof by contrapositive? Contradiction? Direct proof? It should be clear after the first sentence what the author's *goal* is. Write down the goal of the proof.
 - You've identified the structure. Now it's detail time. The proof will be written as a series of mathematical clauses or phrases. After each phrase, STOP.
 - Ask, "Is that right? Does that follow? Why is that true?"
 - If you don't immediately find an answer ("Because that's the definition," "Because that's what Lemma 1 said," etc) it's time to go back to your pad of paper.
 - For example, the author might write on line 3, "It follows that n is the product of at least three distinct primes." Why should this be true? Write down everything you know about n from before line 3. If n were prime or the product of two distinct primes, could we be where we are?
 - Once you've understood this claim (*and only then*), move to the next clause. Read it, understand what it says, and STOP.
 - Why should this be true? Is that a legal move? Is this right? Write it down! Check for yourself!
 - Continue in this way until the end of the proof.
8. Continue in this way until the end of the section, with every definition, example and theorem. Phrases like "it is easy to see that," "it follows that," "a short calculation shows that," and "the reader can check that" all mean: STOP. Write it down! Check for yourself!